# A generalized trigonometric series function model for determining ionospheric delay*

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**Abstract** A generalized trigonometric series function (GTSF) model, with an adjustable number of parameters, is proposed and analyzed to study ionosphere by using GPS, especially to provide ionospheric delay correction for single frequency GPS users. The preliminary results show that, in comparison with the trigonometric series function (TSF) model and the polynomial (POLY) model, the GTSF model can more precisely describe the ionospheric variation and more efficiently provide the ionospheric correction when GPS data are used to investigate or extract the earth's ionospheric total electron content. It is also shown that the GTSF model can further improve the precision and accuracy of modeling local ionospheric delays.

**Keywords:** global positioning system (GPS), ionospheric delay, total electron content (TEC).

The precision of the calculated regional or global ionospheric delays based on global positioning system (GPS) data depends to a large degree on the modeling effectiveness of the corresponding local ionospheric delays over the coverage areas[1~14]. Many scientific studies, especially the single-frequency-radio users, need high precision ionospheric delay information, i.e. high precision determination of the corresponding local ionospheric delays[11~14]. Therefore, it is necessary to choose an appropriate ionospheric delay model. So far, the polynomial (POLY) model is the most frequently used model in the fields of GPS research and application[15~18]. However, the POLY model can only provide an ideal precision during a short session of about several hours. Georgiadous[19] improved the local ionospheric delay modeling by constructing a new model based on a set of trigonometric series functions (TSF). However, the TSF model is also limited in describing characteristics of the local ionospheric variations, since fixed parameters are used in the model. To further improve the model performance, this paper proposes a new model called the generalized trigonometric series function (GTSF) model, which consists of a set of trigonometric series functions in a generalized way in the geomagnetic frame. Using a set of multi-day GPS data, we compare the fitted precisions of the ionospheric delays obtained by the GTSF model, the TSF model, and the POLY model, respectively. Advantages of using the GTSF and single frequency GPS data to precisely extract and investigate the ionospheric delays are verified.

## 1 The GTSF model

It is a crucial step for establishing a high precision vertical ionospheric delay correction model to efficiently describe the properties of diurnal ionospheric variations. Therefore, ionospheric delays are usually determined by using high precision dual frequency data of a session of one day when fitting the ionospheric delay based on the GPS data. The diurnal behavior of the total electron contents (TECs) depends strongly on the season, latitude, solar and geomagnetic activity, etc. Due to the large differences between the diurnal behaviors of the earth's TEC over different times or locations, it is very difficult to find a unified expression that can be effectively used in any local area. Thus, it is difficult to establish a commonly used ionospheric model.

However, as we know, most of the GPS users are in the middle-latitude areas. In these regions, the vertical ionospheric total electron content (VTEC) over a single station or a local area generally shows the characteristics of diurnal variations as follows: at daytime, the variation of VTEC can be approximately illustrated in cosine function and the maximum of the daily TEC usually occurs at the local time of $t = 14$ h;

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at nighttime, the ionospheric variation is in a relatively stable state and has comparatively small magnitudes, and the ionospheric variation does not seem to change with the local time $t$ during this period. Hence, if $\phi_m$ and $t_{sip}$ denote the magnetic latitude and the local time of the sub-ionospheric point (SIP), respectively, and $h = 2\pi(t_{SIP} - 14)/T$ ($T = 24$ h), then the diurnal variation of the VTEC can be approximately considered to be the combination of the following terms: $A_1$: the combined variations of the related factors except for that of the local times and latitudes; $\sum_i^{N_2}\{A_i\phi_m^i\}$: the variations related to the latitudes only; $\sum_i^{N_3}\{A_ih^i\}$: the variations related to the local times only; $\sum_{i=1,j=1}^{N_I,N_J}\{A_i\phi_m^ih^j\}$: the combined variations related to the latitude and the local time; $\sum_{i=1}^{N_4}\{A_{i1}\cos(ih)+A_{i2}\sin(ih)\}$: the combined effects of the periodic variations related to the local time.

Here, $A_i$ are the unknown VETC parameters to be determined. If the sequence number $i$ of the $A_i$ is adjusted according to the characteristics of ionospheric diurnal variation with an approximate period of 24 h under the normal conditions over the areas of the middle latitudes, then an ionospheric model can be constructed based on the trigonometric series functions as follows:

$$
\begin{aligned}
f_{vtec} = A_1 &+ \sum_{i=1}^{N_2}\{A_{i+1}\phi_m^i\} + \sum_{i=1}^{N_3}\{A_{i+N_2+1}h^i\} \\
&+ \sum_{i=1,j=1}^{N_I,N_J}\{A_{i+N_2+N_3+1}\phi_m^ih^j\} \\
&+ \sum_{i=1}^{N_4}\{A_{2i+N_2+N_3+N_I+N_J-1}\cos(ih) \\
&+ A_{2i+N2+N3+N_I+N_J}\sin(ih)\},
\end{aligned} \tag{1}
$$

where $\phi_m = \phi_g + 0.064\cos(\lambda_g - 1.617)$, $\phi_g$ and $\lambda_g$ are the geographic latitude and geographic longitude of SIP, respectively.

If $\alpha_i = C_1 \circ A_i$ ($C_1 = 40.3/f_1^2$), then Eq. (1) will be changed into the corresponding vertical ionospheric delay model:

$$
\begin{aligned}
I_{1,v} = \alpha_1 &+ \sum_{i=1}^{N_2}\{\alpha_{i+1}\phi_m^i\} + \sum_{i=1}^{N_3}\{\alpha_{i+N2+1}h^i\} \\
&+ \sum_{i=1,j=1}^{N_I,N_J}\{\alpha_{i+N2+N3+1}\phi_m^ih^j\} \\
&+ \sum_{i=1}^{N_4}\{\alpha_{2i+N2+N3+N_I+N_J-1}\cos(ih) \\
&+ \alpha_{2i+N2+N3+N_I+N_J}\sin(ih)\},
\end{aligned} \tag{2}
$$

where $a_i$ ($i = 1, 2, \cdots, n$) are the unknown parameters of the ionospheric model. The unknown parameters in Eq. (2) can be determined by the GPS observations and least squares technique. Georgiadous[19] once proposed a method of using dual frequency GPS carrier phase data to estimate the ionospheric delay based on TSF with fixed parameters, and setting $N_2 = 0$, $N_I = N_J = 1$, $N_3 = 2$, $N_4 = 6$ for Eq. (2). Obviously, by adjusting and selecting different types of model parameters in Eq. (2) expressed in a generalized way, the different local ionospheric properties can be comparatively well described. In theory, the precision of ionospheric TEC can be improved by using the GTSF model with the GPS data obtained at a single station or over a local network. To verify this, the calculation in this paper is only conducted in the geographic reference frame. In fact, the estimated TEC in the solar-geographic frame are very close to those obtained in solar-geomagnetic frame when using the GPS data with sub-daily or higher resolution. From this point of view, it is also suggested that the main factor that generates the ionosphere is not the geomagnetism but the sun.

## 2 Experimental method and the preliminary analyses

### 2.1 GPS data and the basic experimental methods

In this experiment, we compared the effectiveness of the GTSF, TSF, and POLY model to fit ionospheric delays, with a set of dual frequency GPS data obtained at the IGS station, WTZR with known precise coordinates, over the year 2000. On the basis of one day a week, the selected days are as follows: 2, 9, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, 85, 92, 99, 106, 113, 120, 127, 134, 141, 148, 155, 162, 169, 176, 183, 190, 197, 204, 211, 218, 225, 232, 239, 246, 253, 260, 267, 273, 280, 287, 294, 301, 308, 315, 322, 329, 336, 343, 350, 357, 364. One day is considered as one GPS session in calculating. The data interval is 30 s. The elevation cut off is 25°. In addition, the GPS data of the seven (from 30/08/1998 to 05/09/1998) and two consecutive days (days of 138 and 139 of 1997) received at BJFS and the Wuhan station are used respectively to analyze in detail the properties of the ionospheric delays fitted by the GTSF model. In this paper, all TEC values are expressed by ionospheric delays in L1 signals.

In the fields of GPS research and applications, usually the ionosphere is approximated by a thin layer, i. e. an ionospheric spherical shell at a height of $H_{ipp}$ (e. g. 350 km) above the earth's surface[1, 4, 8 ~ 18]. The vertical TEC is parameterized exclusively by a SLM (slim layer model) which refers to a solar-geomagnetic (or solar-geographic) frame. For the IBs (Instrumental Bias) of stations and satellites in their GPS observations, different daily constants are assumed according to different satellites. The reduction between slant and vertical TEC is conducted by a reasonable mapping function and an appropriate ionospheric delay model. The unknown model parameters of $I_{1,v}$ are fitted with the least square technique from the phase smoothed geometry free code observations. The above procedure is considered to be an ideal approach to determining the ionospheric delay with high precision[12]. Such a method is also applied in this paper. Firstly, we assume that the ionospheric layer's height $H_{ipp}$ is 350 km and the earth's radius $R_e$ is 6371 km. A trigonometric SLM mapping function is used for the selected cut off elevation angle of 25°[17]. Secondly, the three different estimation values of $I_{1,v}$ fitted by GTSF, TSF, and POLY are used to correct the selected GPS P1/CA code observations, respectively. Thirdly, dual frequency ionosphere-free code data P3 and the three different types of corrected GPS P1/CA code observations are used to determine the positions of WTZR with single point positioning techniques, respectively. Finally, the four different absolute positioning precisions are compared to analyze the properties of the three models in fitting ionospheric delays and describing other related characteristics of the ionosphere.

## 2.2 Experimental results and analyses

Table 1 gives the different average accuracies of single point positioning using the above four kinds of observations with different ionospheric delay corrections during the whole year, the second half of the year, and a few months of the year 2000 at WTZR, respectively.

It can be seen from Table 1 that although the POLY model has a smaller number of parameters, its ionospheric delay correction precision is lower than those provided by TSF and GTSF. TSF has a relatively high precision but a larger number of parameters. The correction precision of the ionospheric delays fitted by the GTSF model is closer to the self-correction one by forming the ionosphere-free observations P3 and is also obviously higher than that of the fitted ionospheric delays by the TSF model. Table 1 also shows that GTSF has not only a higher precision but also a smaller number of model parameters in handling the ionospheric delays. This is because the GTSF model can reasonably select the types and number of the model parameters according to the seasonal ionospheric variations. Hence, the number of the GTSF parameters is obviously smaller and totally close to the number of the POLY model parameters. The number of POLY model parameters is usually suggested to be five by many researchers[15 ~ 18]. The GTSF model may be conveniently applied in the same way as the POLY model. The GTSF model parameters can be selectively adjusted, due to the fact that these parameters can better reflect the characteristics of the time-space variations of the local ionosphere. Although the POLY model parameters can also be adjusted in theory, they cannot efficiently illustrate the ionospheric characteristics. Therefore, the adjustability of the POLY model parameters is quite limited.

Table 1. Comparison of the average precisions of absolute positioning using the corrected P1/CA code GPS observations with the estimated ionospheric delays provided by the POLY, TSF, and GTSF model over the whole year, second half year, and a few months of the year 2000, at the IGS station, WTZR

| Ionospheric delay correction models or methods | Time periods a) (GPS dates) | Positioning precision (m) | Number of model parameters |
|---|---|---|---|
| POLY | Whole year | 2.86 | 5 |
| | Half year | 0.92 | |
| | 118 days | 1.02 | |
| TSF | Whole year | 2.77 | 15 |
| | Half year | 0.82 | |
| | 118 days | 0.95 | |
| GTSF | Whole year | 2.65 | 5 ~ 6 (Average numbers) |
| | Half year | 0.69 | |
| | 118 days | 0.78 | |
| P3 (Self-correction method) | Whole year | 2.61 | None |
| | Half year | 0.60 | |
| | 118 days | 0.65 | |

a) The whole year: from GPS day of 2 to 364/2000; the half year: from GPS day of 183 to 364/2000; the 118 days: from GPS day of 183 to 301/2000.

Fig. 1 shows the continuous diurnal variations of the vertical ionospheric delays fitted by the GTSF model and the POLY model using the dual frequency phase GPS data received at the BJFS station during one week. Comparing Fig. 1 (a) with Fig. 1 (b), we find that although the estimated vertical ionospheric delays calculated by the two models can illustrate the basic diurnal variations, the results in Fig. 1 (b) provided by the POLY model change too smoothly, and accordingly cannot resolve the variations of the TEC and have relatively bad continuities between the days. In comparison, the results fitted by the GTSF model can more effectively illustrate continuities and some other real activities including the small fluctuations of TEC.

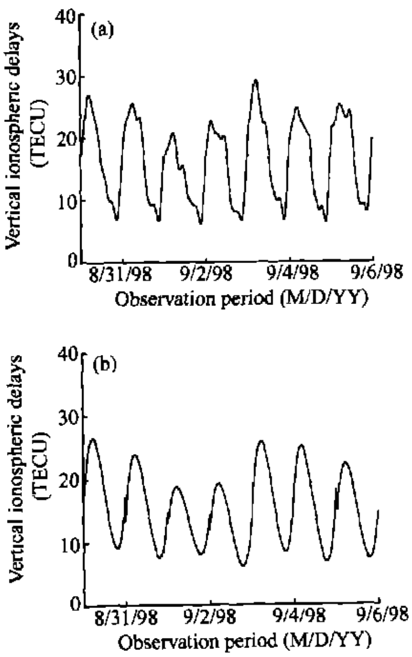


Fig. 1. Diurnal variations of the estimated vertical ionospheric delays fitted by the (a) GTSF model and (b) POLY model using the dual frequency GPS data. Time intervals: from 30/8/1998 to 05/9/1998; location: BJFS station.

Fig. 2 shows the estimated ionospheric delays fitted by the GTSF model using the GPS data over the two consecutive days obtained at a station in Wuhan. The data was individually processed for each day. It can be seen that GTSF can reflect the periodic variation of the stable ionosphere well. In addition, it is shown that the GTSF model can not only describe the deterministic correlations of the local ionospheric delays but also give a comparably good prediction for the variation of the local deterministic vertical ionospheric delays.

Usually, in the middle-latitude areas, the diurnal variations of the ionospheric VTEC monitored by a single station or a small GPS network have the obvious periodic characteristics. The frequently used local

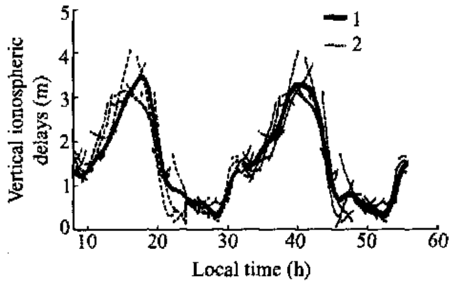


Fig. 2. Variations of the estimated vertical ionospheric delays using the dual frequency GPS data and GTSF model. 1, The vertical ionopheric delays at the reference station; 2, the vertical ionospheric delays at the different IPPs (ionospheric pierce points). Time intervals: the days of 138 and 139 for the year 1997; location: Wuhan station.

ionosphere model, POLY, is usually set up in a solar-geographic frame based on the assumption that the ionosphere is in an approximate static state. Hence, the POLY model can usually provide higher precision ionospheric delay corrections during a period of only about several hours. If the POLY model is used to get the diurnal ionospheric variation with a high precision, a whole day must be divided into 6 ~ 8 sub-intervals, which will bring some limits to the precision in modeling the ionospheric delays when a long period (e.g. one day) of GPS data is used. This is due to the fact that the continuities of the estimated ionospheric delays between the different sub-intervals in a day cannot be guaranteed in theory. Therefore, it is required to efficiently fit the ionospheric delay with a high enough precision, and a mathematical function model in the earth-fixed reference system should be constructed to efficiently describe the behaviors of the vertical ionospheric delay with respect to the local time $t$. It can also be seen from the experimental results that, comparing the TSF model with the frequently used POLY model in the fields of GPS research and application, the GTSF model is better in describing and reflecting the characteristics of the ionospheric variations and in fitting the ionospheric delays. It is desirable to use the GTSF model to improve the performance of describing the ionospheric activities as well as to provide better ionospheric delay corrections for many types of single frequency radio users (including GPS users).

## 3 Conclusions and future work

In this paper, an ionospheric model is proposed and its application in extracting the ionospheric delay with high precision based on the generalized trigonometric series function (GTSF) has been investigated.

A high precision method which can determine the local ionospheric delays is discussed and analyzed. The method includes the following steps: select GTSF to describe the variation of the ionospheric delays, consider the inter-frequency bias to be a systematic error, use a reasonable mapping function to efficiently separate the ionospheric delay with the inter-frequency bias, and estimate the ionospheric model parameters by using least squares method from the phase (or phase smoothed code) GPS data. Experimental results have shown that such a method is an efficient and reliable approach to determining the ionospheric delays. However, results given in this paper are preliminary, and more experiments need to be conducted to further improve and verify GTSF. Some crucial issues of the GTSF model need to be further investigated, especially the internal relations and mechanisms between the selection of the model parameters, the distribution patterns of different local ionospheres, and the characteristics of their time-space variations.

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